Polynomials

Maths

Class: 9

By: Manish Gupta

01 Recap Of Algebra till Class 8

(To watch Video for this topic click the link)

https://youtu.be/XWeMjxQtPT0

Algebra: The branch of mathematics which deals with the study of variables and methods of manipulating them.

Variables: Alphabets, Symbols or literals used to represent any unknown quantity. Like 'x', 'y', 'z' $\propto, \beta, \gamma$ etc.

Constants: The fixed Numbers having a specific value, which never changes.Like 5, 7, $\frac{5}{3}$ etc.

Algebraic Terms: they are formed by the operations of multiplication and/or division between constants and variables.Like $2x, \frac{x}{2}, \frac{4}{x}, \frac{4x}{5y}, \frac{7x}{3}$

Algebraic Expression: A mathematical statement involving addition and/or subtraction of one or more terms. Like $3x^2, 5x^2 + 5x, 7x + 5y$ etc.

We classify Algebraic Expressions as Monomials, Binomials, Trinomials and Expressions having more than three terms.

Degree of an Algebraic Expression: The Highest power of any term in an Algebraic Expression. Like in Expression $4x^5 + 7x^3 + 2x + 3$ the highest power of variable x is 5 and thus the degree of this expression is 5.

Likewise in the expression $7x^2y^3 - 4xy^2 + 8x^3y^3$ the highest power of any term is of third term that is 6 thus the degree of this expression is 6.

02 Polynomials And Its Standard Forms:

(To watch Video for this topic click the link)

https://youtu.be/SnEliwxilNc

Polynomials are those particular algebraic expressions in which the powers to which the variables are raised are whole numbers. Like $4x^3 + 3x^2 - 9x + 7$ is a polynomial but $6x^{\frac{1}{2}} + 7x^2 - 4x - 9$ is not a polynomial because the first term of the algebraic expression has a rational number power of the variable which is not a whole number.

Polynomials may be in one or more variables: Like $4x^2y + 7xy^2 - 3y^3 + 8x^3$ is a polynomial in two variables x and y we write it as

p(x,y) = $4x^2y + 7xy^2 - 3y^3 + 8x^3$ read as polynomial in x and y.

A polynomial in a single variable may be written as p(x) = $4x^3 - 9x^4 + 7x^5 - 3x - 4$

Standard Form Of writing The Polynomials: If all the terms of a polynomial are written in decreasing order of their degrees then the polynomial is said to be written in standard form. Like p(x) = $9x^3 - 4x^2 + 7x^5 - 4x^7 + x^8$ is not in descending order of the degree of its term if we arrange in standard form it will be written as

p(x) = $x^8 - 4x^7 + 7x^5 + 9x^3 - 4x^2$

03 Types Of Polynomials Based On Their Degree:

(To watch Video for this topic click the link)

https://youtu.be/GqSck52jjWk

Actually there are many types but the major ones we are interested in class 9 are polynomials in a single variable. They may be classified as Linear, Quadratic, Cubic Polynomials and of higher degree.

Linear Polynomials: A Polynomial having Degree one like $p(x) = 3x + 7$ is called a linear polynomial because if we draw a graph between p(x) at y- axis and x at x- axis then we get a straight line. The general form of a linear polynomial is p(x)= ax+b, where a and b are any real nos.

Quadratic Polynomial: A Polynomial having degree two is called Quadratic Polynomial. Its graph between p(x) and x is a curve which may intersect x-axis in maximum two points. The general form of a quadratic polynomial is $ax^2 + bx + c$ where a,b,c are any real nos.

Cubic Polynomial: A Polynomial having degree three is called cubic Polynomial. Its graph between p(x) and x is a curve which may intersect x-axis in maximum three points. The general form of a quadratic polynomial is $ax^3 + bx^2 + cx + d$ where a,b,c and d are any real nos.

04 Value Of A Polynomial For A Given Value Of Variable

(To watch Video for this topic click the link)

https://youtu.be/L8s7pv7xc_k

For finding the value of a polynomial in one variable we replace the variable by the given value and then simplify it. For example

Find the value of the polynomial $x^3 + 3x^2 + 5x - 14$ at $x = 2$

We put $p(x) = x^3 + 3x^2 + 5x - 14$

And then we replace x by 2 to find p(2) i.e

$p(2) = 2^3 + 3 \times 2^2 + 5 \times 2 - 14$

$= 8 + 3 \times 4 + 10 - 14$

$= 8 + 12 + 10 - 14 = 16$

Zeroes of a Polynomial

If the value of any polynomial p(x) for any value of x=a is zero, then x=a is called the zero of polynomial p(x). thus we define the zero of a polynomial as follows.

The value of the variable at which the value of the polynomial becomes zero is called the zero of the polynomial.

For example let us consider p(x)= $x^2 + 9x + 14$

If we find $p(-2)$ and $p(-7)$ then

$p(-2) = (-2)^2 + 9 \times (-2) + 14$

$=4 - 18 + 14$

$=0$

And $p(-7) = (-7)^2 + 9 \times (-7) + 14$

$= 49 - 63 + 14$

$=0$

Thus $p(-2)$ and $p(-7)$ both are zero thus we say that $x = -2$ and $x = -7$ are the two zeroes of this polynomial. Also we observe that at any other value of x the value of the polynomial is not zero. Thus we arrive at the following important conclusion.

A polynomial p(x) having degree n has maximum n zeroes i.e the number of zeroes of a polynomial can never exceed its degree.

05 Division of Polynomials and Remainder Theorem

(To watch Video for this topic click the link)

https://youtu.be/SY9ioBV3t-8

For dividing a polynomial by any linear polynomial the following steps are followed.

Step 1: Write both the divisors and dividend in standard form(in decreasing order of degrees of terms)

Step 2: Obtain the firs term of the quotient by dividing highest degree term of dividend by that of divisor.

Step 3: Multiply the so obtained term with the divisor and subtract it from the dividend. Normally we get the new dividend which is one degree lesser.

The same steps are repeated till we get the final remainder having degree less than the divisor. For Example:

Divide $4x^3 - 6x^2 + 3x - 9$ by $x - 1$

$x - 1)4x^3 - 6x^2 + 3x - 9(4x^2 - 2x + 1$

$4x^3 - 4x^2$

$- \quad +$

$0 - 2x^2 + 3x - 9$

$-2x^2 + 2x$

$+ \quad -$

$0 + x - 9$

$x - 1$

$- \quad +$

$0 - 8$

This way we see that when the division is carried out the remainder so obtained is -8

Remainder Theorem: This theorem gives us a way of finding the remainder obtained without the long division process. The simplest way to understand the theorem is as follows.

If we have to find the remainder obtained on dividing any polynomial by any linear polynomial, then we first find the zero of the divisor (the linear polynomial). We put this value in place of x in the dividend. The value so obtained is the remainder.

In short it may be stated as:

If a Polynomial $p(x)$ is divided by a linear polynomial $x - a$ then the remainder obtained on division is $p(a)$

In the above division example, the divisor was -1 . for finding its zero we equate it to 0 i.e put $x - 1 = 0$ i.e $x = 1$ is a zero of the divisor. Next if we find the value of the dividend at $x = 1$

$p(1) = 4(1)^3 - 6(1)^2 + 3(1) - 9$

$p(1) = 4 - 6 + 3 - 9 = -8$

06 Factor Theorem:

(To watch Video for this topic click the link)

https://youtu.be/VthErv9Lyyk

The factor theorem gives us a method for finding the factors of a polynomial without applying the long division method. It is like an extension of remainder theorem. As we know we call a linear polynomial a factor of a polynomial of degree n if the division is such that the remainder is zero. Thus applying this fact we state Factor Theorem as follows:

If $x - a$ is a factor of a polynomial $p(x)$ then $p(a) = 0$

That is if $x - a$ is a factor of p(x) then the value of $p(x)$ at the zero of $x - a$ is zero.

Important Note: We had defined the zero of a polynomial as the value of the variable at which the value of the polynomial becomes zero. Also we say that at $x = a$ the value of polynomial is zero then $x - a$ is a factor of the polynomial. Thus we have an important relation between the zeroes of polynomial and its factors. We state it as follows.
If $x = a$ is a zero of $p(x)$ then $x - a$ is a factor of $p(x)$
Also it may be noted that the zeroes of a polynomial never changes if it is multiplied or divided by any constant real number.

07Algebraic Identities:

(To watch Video for this topic click the link)

https://youtu.be/sSv2909ZznA

We have studied the following four identities in class 8

1. $(x+y)^2 = x^2 + 2xy + y^2$
2. $(x-y)^2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x+y)(x-y)$
4. $(x+a)(x+b) = x^2 + (a+b)x + ab$

Now we have the following important identities:

5. $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

For example:

Write in expanded for using a suitable identity:

$(2x+3y+4z)^2$

Here we see that in place of x we have 2x, in place of y we have 3y and in place of z we have 4z. thus we use the identity as follows:

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(2x+3y+4z)^2 = (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(3y)(4z) + 2(4z)(2x)$

$(2x+3y+4z)^2 = 4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16zx$

While expanding we have just put the values of x, y and z in the identity. Similarly

$(a-3b+2c)^2 = (a)^2 + (-3b)^2 + (2c)^2 + 2(a)(-3b) + 2(-3b)(2c) + 2(2c)(a)$

(Here as we observe that in place of x we have a, in place of y we have -3b and in place of z we have 2c). we have to put the terms for x,y and z along with there signs.

Thus, $(a-3b+2c)^2 = a^2 + 9b^2 + 4c^2 - 6ab + 12bc + 4ca$

The 6^{th} and the 7^{th} identities are for cubes as follows:

6. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and
7. $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

For example: Expand $(2a+3b)^3$ using a suitable identity:

Here we see that in place of x we have 2a and in place of y we have 3b, thus we can expand by putting the values as

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ so

$(2a+3b)^3 = (2a)^3 + 3(2a)^2 \times 3b + 3 \times 2a \times (3b)^2 + (3b)^3$

$(2a+3b)^3 = 8a^3 + 36a^2b + 54ab^2 + 27b^3$

07 Solutions To NCERT Examples and Exercises can be viewed at the following links:

Exercise 2.1 NCERT	https://youtu.be/nGONO6OXROA
Exercise 2.2	https://youtu.be/PdUVg-Alavw
NCERT Examples Remainder Theorem	https://youtu.be/1f_0zTfJIT0
Exercise 2.3 polynomials Class 9	https://youtu.be/ppxIbA54FO4
NCERT Exercise 2.4 Solutions	https://youtu.be/Kvs3Piin51o
Exercise 2.5 Q 1 to Q 5	https://youtu.be/y7KCo1GVaZA
Exercise 2.5 Q 6 to Q 10	https://youtu.be/RV36qFcWHB4
Exercise 2.5 Q. 11 to Q. 16	https://youtu.be/THVOC6XFXS8
Some Extra Questions From The Chapter	https://youtu.be/vrKJ1uXe1mM

Worksheet 1

1. Write whether the following statements are **True** or **False**. Justify your answer.

(i) A binomial can have at most two terms

(ii) Every polynomial is a binomial

(iii) A binomial may have degree 5

(iv) Zero of a polynomial is always 0

(v) A polynomial cannot have more than one zero

(vi) The degree of the sum of two polynomials each of degree 5 is always 5.

2. Find the value of a, if $x - a$ is a factor of $x^3 - ax^2 + 2x + a - 1$.

3. For the polynomial $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$ **write,**

(i) the degree of the polynomial

(ii) the coefficient of x^3

(iii) the coefficient of x^6

(iv) the constant term

4. If p(x)= $x^2 - 4x + 3$, evaluate ; $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

5. By Remainder Theorem find the remainder, when $p(x)$ is divided by $g(x)$, where

$p(x) = x^3 - 2x^2 - 4x - 1, \quad g(x) = x + 1$

6. Show that : $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.

Worksheet 2

1. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$ find the value of a.

2. Factorise the following by using splitting the middle term method:

(i) $x^2 + 9x + 18$

(ii) $6x^2 + 7x - 3$

(iii) $2x^2 - 7x - 15$

3. Factorise the following by using factor theorem:

(i) $2x^3 - 3x^2 - 17x + 30$

(ii) $x^3 - 6x^2 + 11x - 6$

4. If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$

5. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that:

$a^3 + b^3 + c^3 - 3abc = -25$